HOSSAM GHANEM

(51) 6.3 Volumes "General"

كيف تختار طريقة الحل

	$y = f(x)$, $y = g(x)$	$x = f(y)$, $x = g(y)$
الدوران حول $x - axis$ أو ما يوازيه	W.M	C.S
الدوران حول $y - axis$ أو ما يوازيه	C.S	W.M

Example 1 36 January 17, 2010	Set up an integral for the volume of the solid generated by revolving the region bounded by $y = x^2 + 4$, $y = 1$, $x = 0$ and $x = 2$ about: (a) the line $y = -2$ (b) the line $x = 4$.

Solution

$y = x^2 + 4$, $y = 1$

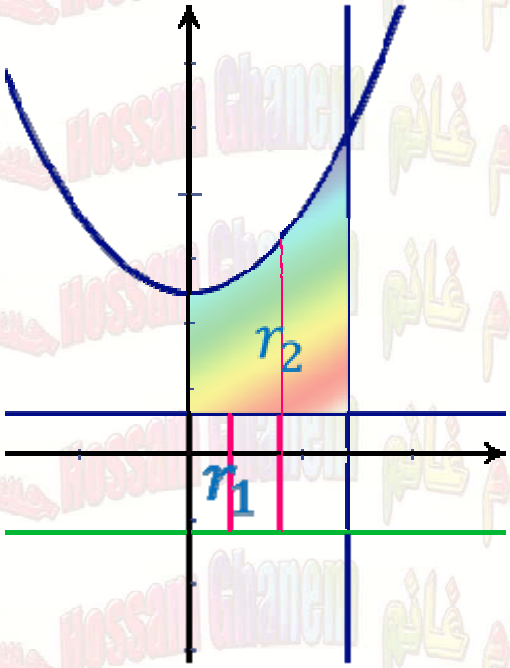
(a) the line $y = -2$

$r_2 = (x^2 + 4) + 2$

$r_1 = 2$

$$V = \pi \int_0^2 [r_2{}^2 - r_1{}^2] \, dx$$

$$V = \pi \int_0^2 [(x^2 + 4 + 2)^2 - (1 + 2)^2] \, dx$$



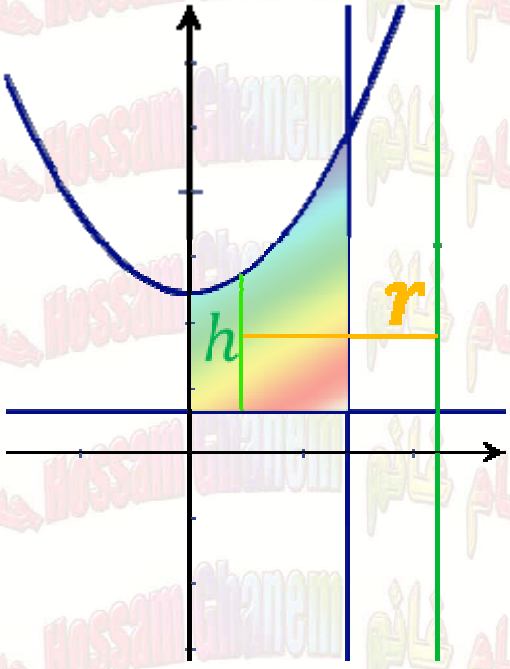
(b) the line $x = 4$

$h = y_2 - y_1 = (x^2 + 4) - 1 = x^2 + 3$

$r = 4 - x$

$$V = 2\pi \int_1^2 rh \, dy$$

$$V = 2\pi \int_0^2 [(4 - x)(x^2 + 3)] \, dx$$



Example 2 33 January 20, 2009	The region bounded by the curves $y=\sqrt{x}$ and $y=x^3$ is revolved about : (a) the line $y=2$, (b) the line $x=2$. Set up an integral that can be used to find the volume of the resulting solid in each case.

Solution

$y=\sqrt{x}$, $y=x^3$

$x^3=\sqrt{x}$

$x=0$, $x=1$

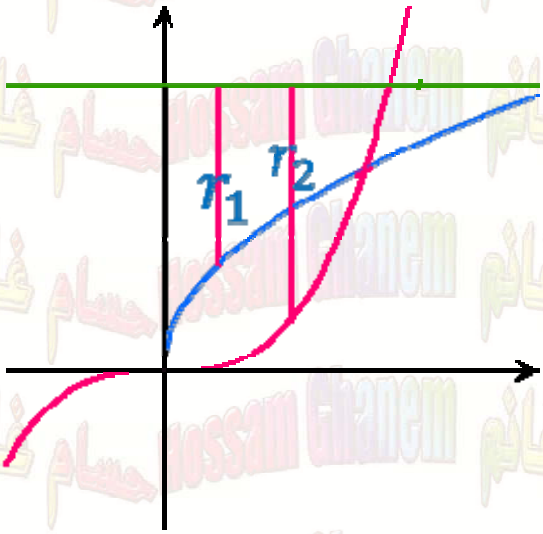
(a) the line $y=2$

$r_2=2-x^3$

$r_1=2-\sqrt{x}$

$$V=\pi\int_0^2[r_2{}^2-r_1{}^2]\ dx$$

$$V=\pi\int_0^1(2-x^3)^2-\left(2-\sqrt{x}\right)^2dx$$

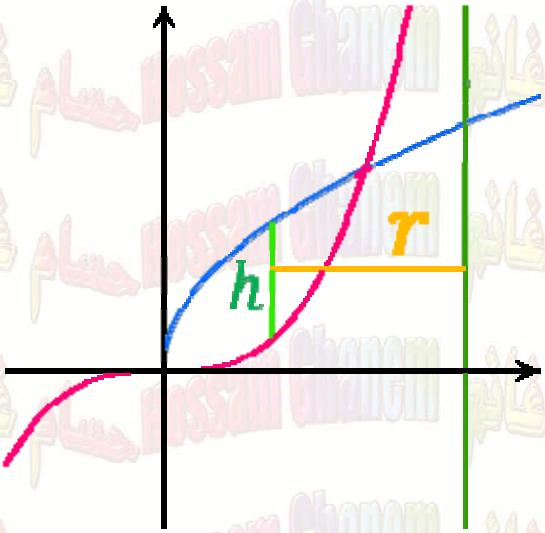


$h=y_2-y_1=\sqrt{x}-x^3$

$r=2-x$

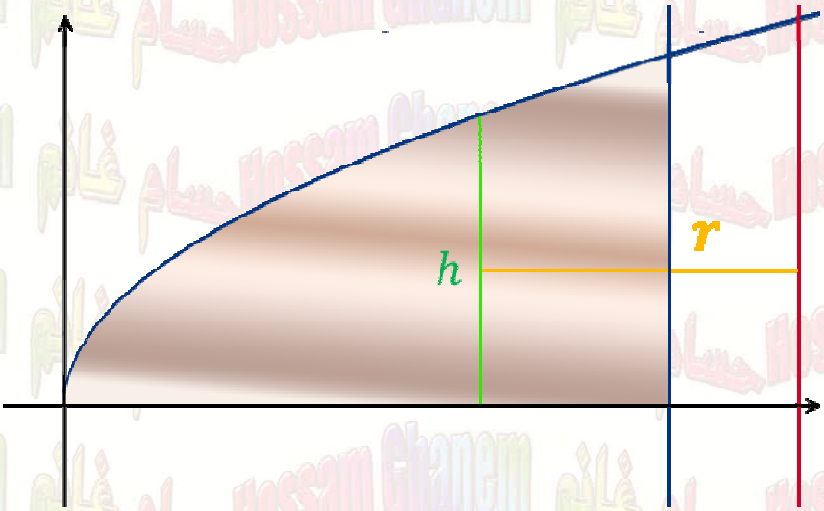
$$V=2\pi\int_1^2 rh\ dy$$

$$V=2\pi\int_0^1(2-x)\left(\sqrt{x}-x^3\right)\ dx$$



Example 3 39 5 June, 2011	[4 pts.] Find the volume of the solid obtained by the rotating the region bounded by $y = \sqrt{x}$, the $x-$ axis and $x = 4$ about the line $x = 5$

Solution



$h = y_2 - y_1 = \sqrt{x} - 0$

$r = 5 - x$

$$V = 2\pi \int_1^2 rh \ dy$$

$$V = 2\pi \int_0^4 (5-x)\left(\sqrt{x}\right) \ dx$$

$$V = 2\pi \int_0^4 x^{\frac{1}{2}}(5-x) \ dx = 2\pi \int_0^4 5x^{\frac{1}{2}} - x^{3/2} \ dx = 2\pi \left[5 \cdot \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{5/2}\right]_0^4$$

$$= 2\pi\left(\frac{10}{3}(8) - \frac{2}{5}(32) - 0\right) = 2 \cdot 8\pi\left(\frac{10}{3} - \frac{2}{5}(4) - 0\right) = 16\pi\left(\frac{50-24}{15}\right) = 16\pi\left(\frac{26}{15}\right) = \frac{416\pi}{15}$$

Homework

1	23 May 26. 2002 Let R be the region bounded by the graphs of the equation $y = x^2 + 4$, $2x + y = 2$, $x = 0$ and $x = 1$. Find the volume of resulting solid if R is revolved about the line $y = 5$
2	18 May 24 ,2000 Let R be the region bounded by the graphs of $y + x^2 - 2x = 0$, $y + 2x = 0$ Set up an integral that can be used to find the volume of the solid generating by revolving R about the line $y = 1$
3	Let R be the region bounded by the graphs of $y + x^2 - 2x = 0$, $y + 2x = 0$ Set up an integral that can be used to find the volume of the solid generating by revolving R about the line $y = 1$
4	20 January 3 ,2001 Let R be the region bounded by the graphs of the equation $y = x^2 + 3$, $y = 0$, $x = 0$ and $x = 2$ Set up an integral that can be used to find the volume of the solid generating by revolving R about the line the line $x = -5$
5	40 August 7, 2011 (3 Points) Set up an integral for the volume that is obtained by revolving the region enclosed between the curves $y = x^2 - 5x$ and $y = x$ about the lines $x = -1$
6	23 May 26. 2002 Let R be the region bounded by the graphs of the equation $y = x^2 + 4$, $2x + y = 2$, $x = 0$ and $x = 1$ Set up an integral that can be used to find the volume of the resulting solid if R is revolving about the line $x = 2$
7	24 May 27. 2001 Set up an integral that can be used to find the volume of the solid obtained by revolving the region bounded by the graphs of the equation $y = 4x - x^2$, and $y = x$ about $x = 3$
8	27 May 30. 2006 The region bounded by the curves $y = \sqrt{x - 1}$, $y = 0$ $x = 5$ is revolved abut the line $x = 7$. Set up an integral that can be used to find the volume of the resulting solid in each case
9	28 January 13. 2007 The region bounded by the curves $y = x^2$, $y = 4$ is revolved abut the line $x = 5$. Set up an integral that can be used to find the volume of the resulting solid in each case

Homework

10	31 June 5, 2008 The region bounded by the curves $y = 4 - x^2$, $y = 0$ is revolved abut the line $x = 3$. Set up an integral that can be used to find the volume of the resulting solid
11	16 June 6 .1996 The region bounded by the graphs of the curves $y = x^2 - 3$ and $y = 5 - x^2$ is revolved about the line $x = 6$. Find the volume of the resulting solid.
12	17 January 8 ,1997 The region bounded by the graphs of $y - x^2 - 3 = 0$, $y - 3x + 1 = 0$, $x = 0$ and $x = 1$ is revolved about the line $x = -1$, Find the volume of the resulting solid
13	26 June 7, 2003 The region bounded by the graphs of the equations $y = \sqrt{x+2}$, $x = 0$, $y = 0$ and $y = 1$ is revolved about the $x - axis$. Set up an integral that can be used to find the volume of the resulting solid.
14	32 August 02, 2008 Set up an integral for the volume of the solid obtained when the region bounded by $y = x^2 + 3$ and $y = 4x$ is revolved about: a) $y - axis$, b) $y = -1$.
15	33 January 20, 2009 The region bounded by the curves $y = \sqrt{x}$ and $y = x^3$ is revolved about : (a) the line $y = 2$, (b) the line $x = 2$. Set up an integral that can be used to find the volume of the resulting solid in each case
16	41 7 January 2012 [3+3 Pts.] Set up an integral for the volume of the solid obtained by rotating the region enclosed by the curves $y = x^2$ and $y = 1$ about each of the lines : (a) $y = -2$ (b) $x = 2$
17	20 January 3 ,2001 Let R be the region bounded by the graphs of the equation $y = x^2 + 3$, $y = 0$, $x = 0$ and $x = 2$Set up an integral that can be used to find the volume of the solid generating by revolving R about the line $x - axis$

Homework

18	28 January 13. 2007 The region bounded by the curves $y = x^2$, $y = 4$ is revolved abut the line $y = -1$.Set up an integral that can be used to find the volume of the resulting solid in each case
19	27 May 30. 2006 The region bounded by the curves $y = \sqrt{x-1}$, $y = 0$ $x = 5$ is revolved abut the line $y = -3$. Set up an integral that can be used to find the volume of the resulting solid in each case
20	19 July 29, 2000 The region bounded by the graphs of the equation $y = x^2$ and $x = y^2$ is revolved abut the line $y = -3$. Find the volume of resulting solid
21	22 August 11.2001 A The region bounded by the graphs of the equation $y = \sqrt{x}$ and $y = x$ is revolved abut the line $y = -1$. Find the volume of resulting solid
22	The region bounded by the graphs of the equation $y = x^3$, $x = 1$ and the $x - axis$ is revolved abut the line $y = -1$ Set up an integral that can be used to find the volume
23	Set up an integral for the volume of the solid obtained when the region bounded by $y = x^2$ and $y = 4x$ is revolved abut $y = -1$.
24	30 Jan. 12. 2008 Find the volume of the solid obtained by rotating the region bounded by the curves of $y = x$ and $y = x^2$ about the line $y = 2$
25	37 June 6, 2010 Set up an integral for the volume of the solid obtained by rotating the region enclosed between the curves $y = x^2$ and $y = x + 2$ about : (a) $x = 7$ (b) $y = -1$
26	38 January 15, 2011 the region bounded by the curves $y = 4x - x^2$ and $y = 3$ revolved about : (a) the line $x = 1$ (b) the line $y = -1$ Set up an integral that can be used to find the volume of the resulting solid in each

Homework

27	31 June 5, 2008 The region bounded by the curves $y = 4 - x^2$, $y = 0$ is revolved abut the line $y = -2$. Set up an integral that can be used to find the volume of the resulting solid
28	35 August 15, 2009 The region bounded by the curves $y = x^2$ and $y = x + 2$ is revolved about: (a) the line $y = -1$ (b) the line $x = 3$. Set up an integral that can be used to find the volume of the resulting solid in each case
29	34 June 21, 2009 Set up an integral for the volume of the solid generated by revolving the region bounded by $y = x^2$ and $y = 1$ about: (a) the line $y = -1$. (b) the line $x = 5$.